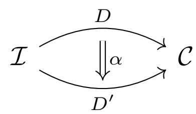
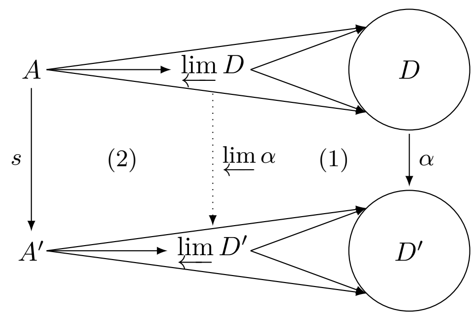
Let $\mathcal{C}$ be a category, $\mathcal{I}$ a small category and D, D' two diagrams $\mathcal{I} \to \mathcal{C}$, and consider a natural transformation between these two diagrams:



Let

$$\left(\varprojlim D \xrightarrow{p_I} D(I)\right)_{I\in\mathcal{I}} \text{ and } \left(\varprojlim D' \xrightarrow{p'_I} D'(I)\right)_{I\in\mathcal{I}}$$

be limit cones of D and D' respectively.



1. There exists a unique map $\varprojlim \alpha : \varprojlim D \to \varprojlim D'$ such that, for all $I \in \mathcal{I}$, the following commutes:

$$\begin{array}{ccc} \varprojlim D & \xrightarrow{p_I} & D(I) \\ {\scriptstyle \varprojlim \alpha}\downarrow & & \downarrow{\scriptstyle \alpha_I} \\ \varprojlim D' & \xrightarrow[p'_I]{} & D'(I) \end{array}$$

2. Given cones

$$\left(A \xrightarrow{f_I} D(I)\right)_{I\in\mathcal{I}} \text{ and } \left(A' \xrightarrow{f'_I} D'(I)\right)_{I\in\mathcal{I}}$$

and a map $A \xrightarrow{s} A'$ such that, for all $I \in \mathcal{I}$, the following commutes

$$\begin{array}{ccc} A & \xrightarrow{f_I} & D(I) \\ {\scriptstyle s}\downarrow & & \downarrow{\scriptstyle \alpha_I} \\ A' & \xrightarrow[f'_I]{} & D'(I) \end{array}$$

Then the following commutes:

$$\begin{array}{ccc} A & \xrightarrow{\bar{f}} & \varprojlim D \\ {\scriptstyle s}\downarrow & & \downarrow{\scriptstyle \varprojlim \alpha} \\ A' & \xrightarrow[\overline{f'}]{} & \varprojlim D' \end{array}$$

Where $\bar{f}$ is the map originating from the correspondence $\mathrm{Cone}(A, D) \cong \hom_{\mathcal{C}}(A, \varprojlim D)$ and $\overline{f'}$ from $\mathrm{Cone}(A', D') \cong \hom_{\mathcal{C}}(A', \varprojlim D')$.